Mid-Semestral Examination I Semester 2002-2003
B. Math. Hons. II Year
Computer-Oriented Numerical Methods
Date: 23-09-2002 Instructor: B. Sury Time: 9.30 - 11.30 AM

1. Let $\phi(x) = e^{-x/2}$. Start with $x_0 = 0.8$ and use iteration 5 times to produce an approx. for a fixed point for ϕ. The correct value is 0.70346742 upto 8 decimals. Show also that any two consecutive iterates x_n and x_{n+1} contain a fixed point in between.
   Use Newton-Raphson method (again starting with $x_0 = 0.8$) 4 times to solve $f(x) = 0$ where $f(x) = x - e^{-x/2}$. Compare with the above result.

2. Compute $\sqrt[3]{17}$ to an accuracy of $\pm 10^{-7}$ using Newton's method.

3. If $f(x) = \prod_1^n (x - \alpha_0)$ and $\alpha_1, \ldots, \alpha_m (m < n)$ have been already computed, then show that

$$x_{k+1} = x_k - \frac{1}{\frac{f'(x_k)}{f(x_k)} - \sum_{i=1}^{m} \frac{1}{x_k - \alpha_i}}$$

   describes a Newton method for finding the zeros $\alpha_{m+1}, \ldots, \alpha_n$.

4. If $p(x)$ is a polynomial of degree 4 such that $p(0) = p(2) = p(4) = 1$ and $p(1) = p(3) = -1$, find $p(5), p(6), p(7)$ without explicitly writing down $p(x)$.

5. Define $f[x_0, \ldots, x_n]$ for $x_0 \leq x_1 \leq \ldots \leq x_n$ and prove that if $f \in c^n[a, b]$ where $x_i \in (a, b)$, then $f[x_0, \ldots, x_n]$ is a continuous function of each x_i.

6. Show that h can be made nearly as large as $\frac{1}{8}$ so that an equispaced table for $\sqrt{x}(x \geq 1)$ with a cubic poly-interpolation gives 5 place accuracy.

7. Show that the Newton 'pulchernima' quadrature formula for integrating $\int_a^b f(x)dx$ if $[a, b]$ is divided into $3n$ parts and a piecewise cubic

polynomial whose pieces are joined together at the points x_{3i} is used, is:

$$\begin{aligned}\int_a^b f(x)dx &= \frac{3h}{8}(y_0 + 3y_1 + 3y_2 + 2y_3 + \ldots + 3y_{3n-1} + y_{3n}) \\ &\quad -\frac{h^4}{80} f^{(4)}(t) \cdot (b-a) \text{ for some } t \in (a,b)\end{aligned}$$

(when $f \in c^4$).